Zhang's inequality for log-concave functions

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(joint with D. Alonso-Gutiérrez and J. Bernués)

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First definitions

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- For any $M \subset \mathbb{R}^n$, then χ_M is the characteristic of M

$$\chi_M : \mathbb{R}^n \to \{0, 1\} \quad \text{s. t.} \quad \chi_M(x) = \begin{cases} 1 & \text{if } x \in M \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 1 (Isoperimetric ineq. 1915)

Let $M \in \mathcal{C}^1$ and $\overline{M}$ compact. Then

$$n|\mathbb{B}_2^n|^{\frac{1}{n}}\,|M|^{\frac{n-1}{n}} \leq S(M).$$

"=" iff $M = \mathbb{B}_2^n$.

Geometric & Analytic ineqs.

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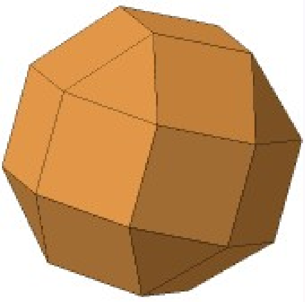
Theorem 2 (Sobolev ineq. '38)

Let $f : M \to \mathbb{R}$ with $M \in \mathcal{C}^1$ compact and $f \in \mathcal{C}^1$. Then

$$n|\mathbb{B}_2^n|^{\frac{1}{n}}\left(\int_{\mathbb{R}^n} |f|^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}} \leq \int_{\mathbb{R}^n} |\nabla f|$$

"$=$" iff $f = \chi_{\mathbb{B}_2^n}$.

- K is a convex body, i.e., a convex compact set of $\mathbb{R}^n$.



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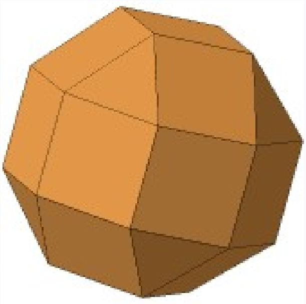

- $\mathcal{K}^n$ set of n-dimensional convex bodies.

- If $x \in \mathbb{R}^n$, then $x^\perp$ is the hyperplane orthogonal to x.

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$$\|x\|_{\Pi^*(K)} := |x||P_{x^\perp} K|.$$

• The product $|K|^{n-1}|\Pi^*(K)|$ is an affine invariant.

Petty projection inequality

Theorem 3 (Petty 1971)

Let $K \in \mathcal{K}^n$. Then

$$|K|^{n-1}|\Pi^*(K)| \leq \frac{\pi^{\frac{n}{2}}\Gamma\left(\frac{n+1}{2}\right)^n}{\Gamma\left(\frac{n+2}{2}\right)^n}.$$

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Petty projection ineq. $\Rightarrow$ Isoperimetric ineq.

Zhang's inequality

Theorem 4 (Zhang 1991)

Let $K \in \mathcal{K}^n$. Then

$$\frac{\binom{2n}{n}}{n^n} \leq |K|^{n-1} |\Pi^*(K)|.$$

"$=$" iff K is an n-simplex.

Polar projection body of f

For every $f \in \mathcal{W}^{1,1} = \{f \in L^1(\mathbb{R}^n) : \frac{\partial f}{\partial x_i} \in L^1(\mathbb{R}^n), i = 1, \ldots, n\}$ with compact support, let $\Pi^*(f)$, the polar projection body of f, be the unit ball of the norm

$$\|x\|_{\Pi^*(f)} := \int_{\mathbb{R}^n} |\langle \nabla f(y), x \rangle| dy.$$

Theorem 5 (Zhang 1999)

Let $f \in \mathcal{C}^1$ with compact support. Then

$$\|f\|_{\frac{n}{n-1}} |\Pi^*(f)|^{\frac{1}{n}} \leq \frac{\pi^{\frac{1}{2}} \Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{n+2}{2}\right)}.$$

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Functional affine inequalities

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$$\|f\|_{\frac{n}{n-1}} \left(\int_{\mathbb{S}^{n-1}} \|\nabla_u f\|_1^{-n} du\right)^{\frac{1}{n}} \leq \frac{n^{\frac{1}{n}}\pi^{\frac{1}{2}}\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{n+2}{2}\right)}.$$

Log-concave functions

- $f : \mathbb{R}^n \to [0, \infty)$ is log-concave if $f(x) = e^{-u(x)}$ for some $u : \mathbb{R}^n \to (-\infty, \infty]$ convex,

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- $\mathcal{F}(\mathbb{R}^n)$ log-concave integrable functions in $\mathbb{R}^n$.

- If $f \in \mathcal{F}(\mathbb{R}^n) \cap \mathcal{W}^{1,1}$ then

$$\|x\|_{\Pi^*(f)} = 2|x| \int_{x^\perp} P_{x^\perp} f(y) dy,$$

where $P_{x^\perp} f(y) = \max_{s \in \mathbb{R}} f(y + sx)$.

Functional affine inequalities

Theorem 6 (Alonso-Gutiérrez, Bernués, G.M. +2018)

Let $f \in \mathcal{F}(\mathbb{R}^n)$. Then

$$\int_{\mathbb{R}^n}\int_{\mathbb{R}^n} \min\{f(y), f(x)\} dydx \leq 2^n n! \|f\|_1^{n+1} |\Pi^*(f)|.$$

"=" iff $\frac{f(x)}{\|f\|_\infty} = e^{-\|x\|_\triangle}$ for a simplex $\triangle \ni 0$.

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Remark

If $f(x) = e^{-\|x\|_K}$ with $K \in \mathcal{K}^n$ then Thm. 6 becomes Thm. 4, i.e.

$$\frac{\binom{2n}{n}}{n^n} \leq |K|^{n-1}|\Pi^*(K)|.$$

Proof of Theorem 6

Definition

Let $f \in \mathcal{F}(\mathbb{R}^n)$. Then

$$K_t(f) := \{x \in \mathbb{R}^n : f(x) \geq e^{-t}\|f\|_\infty\} \quad \forall\, t \geq 0.$$

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Lemma 1

Let $f \in \mathcal{F}(\mathbb{R}^n)$. The covariogram $g : \mathbb{R}^n \to \mathbb{R}$ of f

$$g(x) := \int_0^\infty e^{-t}|K_t(f) \cap (x + K_t(f))|dt$$
$$= \int_{\mathbb{R}^n} \min\left\{\frac{f(y)}{\|f\|_\infty}, \frac{f(y-x)}{\|f\|_\infty}\right\} dy$$

is even and $g \in \mathcal{F}(\mathbb{R}^n)$.

Lemma 2

Let $f \in \mathcal{F}(\mathbb{R}^n)$ and g its covariogram. For every $0 < \lambda_0 < 1$ then

$$2\|f\|_1 \Pi^*(f) = \bigcap_{0<\lambda<\lambda_0} \frac{K_{-\log(1-\lambda)}(g)}{\lambda}.$$

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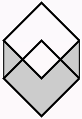

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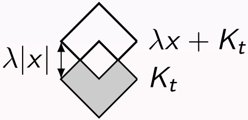

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$$\int_0^\infty e^{-t}\frac{|K_t| - |K_t \cap (\lambda|x|\frac{x}{|x|} + K_t)|}{\lambda} dt \leq |x| \int_0^\infty e^{-t} |P_{x^\perp} K_t| dt$$

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then $x \in K_{-\log(1-\lambda)}(g)/\lambda$ and thus

$$2\|f\|_1 \Pi^*(f) \subset \frac{K_{-\log(1-\lambda)}(g)}{\lambda}$$

for every $0 < \lambda < 1$. $\square$

Proof of Theorem 6

Definition (Ball 1988)

Let $f \in \mathcal{F}(\mathbb{R}^n)$ with $f(0) > 0$. Then

$$\tilde{K}(f) := \left\{ x \in \mathbb{R}^n : n \int_0^\infty f(rx) r^{n-1} dr \geq f(0) \right\}$$

fulfills $\tilde{K}(f) \in \mathcal{K}^n$ and $|\tilde{K}(f)| = \|f\|_1 / f(0)$.

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Lemma 3

Let $g \in \mathcal{F}(\mathbb{R}^n)$ with $g(0) > 0$. If $0 \leq t \leq n/e$ then

$$\frac{t}{(n!)^{\frac{1}{n}}} \tilde{K}(g) \subset K_t(g).$$

"$=$" iff $\frac{g(x)}{g(0)} = e^{-\|x\|_K}$ for some $K \in \mathcal{K}^n$ with $0 \in K$.

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□

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