Conference on Convex, Discrete and Integral Geometry

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Intrinsic volumes on manifolds

Semyon Alesker
Tel Aviv University, Israel

Intrinsic volumes on convex bodies play an important role in convexity since 19th century (Steiner, Minkowski, Alexandrov, Fenchel, Hadwiger...). In 1939 H. Weyl has extended this notion to Riemannian manifolds. Recently the speaker has formulated conjectures on their behavior under Gromov-Hausdorff convergence. The main result of the talk is a proof of one of the conjectures in a very special case.

Longest k-monotone chains

Gregerly Ambrus
Rényi Alfréd Matematikai Kutatóintézet, Hungary

We study higher order convexity properties of random point sets in the plane. Given n uniform i.i.d random points in the unit square, we derive asymptotic estimates for the maximal number of them which are k-monotone position, subject to mild boundary conditions. Besides determining the order of magnitude of the expectation, we also prove strong concentration estimates. We provide a general framework that includes the previously studied cases of $k = 1$ (longest increasing sequences) and $k = 2$ (longest convex chains). These results provide the first step towards extending asymptotic estimates related to convexity properties of random point sets to higher order convexity.

Polarity and some of its relatives

Shiri Artstein-Avidan
Tel Aviv University, Israel

We shall discuss some developments in our understanding of the polarity transform for convex functions, and some new variants of this transform along with their corresponding function classes. In particular we shall explore these in the context of measure transportation.

Spectral gap for some simple convex bodies

Franck Barthe
Université Toulouse, France

Motivated by the KLS conjecture, we investigate Poincaré inequalities for sections of Lp balls, $p \in (1,2)$ and for Orlicz balls. In both cases, we use results by A. Kolesnikov and E. Milman relating spectral gap of a log-concave measure with the one of some level sets of its density. We extend the range of their result on Orlicz balls, thanks to precise volume asymptotics in high dimensions (joint work with P. Wolff). For sections of Lp balls, we study perturbed product measures, and build on the study of Gaussian mixtures initiated by Eskenazis, Nayar and Tkocz (joint work with B. Klartag).

The Weyl principle in pseudo-Riemannian geometry

Andreas Bernig
Goethe–Universität Frankfurt, Germany

The classical Weyl principle states that the coefficients of the volume of a tube around a compact submanifold in euclidean space are invariants of the intrinsic metric. Using the language of valuations and curvature measures on manifolds, they give rise to the intrinsic volumes and Lipschitz-Killing curvature measures. In a recent joint work with D.Faifman (Montreal) and G. Solanes (Barcelona) we extend the theory to pseudo-Riemannian manifolds and more generally to signature changing metrics, where we prove a generalization of the Weyl principle.

Polytopal approximation in the dual Brunn–Minkowski theory

Florian Besau
TU Wien, Austria

In this talk I will present our results for the asymptotic best and randomapproximation of convex bodies by polytopes with a bounded number of verticesor facets, where instead of the volume we are interested in the dual volumes (ordual quermassintegrals) which originate from Lutwak's dual Brunn–Minkowskitheory. This talk is based on joint work together with Steven Hoehner and GilKur; arXiv:1905.08862.

Brunn-Minkowski type inequalities and conjectures

Károly Böröczky
Rényi Alfréd Matematikai Kutatóintézet, Hungary

Given the importance of the classical Brunn-Minkowski inequality, one hopes that the analogues should hold under any reasonable circumstances. However, some recent work by Ludwig, Colesanti and Wannerer shows that this hope can't be fulfilled. The talk discusses what is known in this direction. More considerations are given to the logarithmic Brunn-Minkowski conjecture where some recent exiting developments due to Livshyts, Milman, Kolesanti, Chen, Li, Huang are surveyed indicating the methods.

On the simplex mean width conjecture

Susanna Dann
Universidad de los Andes, Colombia

The simplex mean width conjecture asserts that among all simplices with verticies inthe unit Euclidean sphere, the regular simplex has the largest mean width. In this talkwe suggest a Fourier analytic approach to this question.

This is joint work with Alexander Koldobsky and Dmitry Ryabogin.

The Weyl principle in Finsler geometry

Dimitry Faifman
Université de Montréal, Canada

The Weyl principle in Riemannian geometry states that the intrinsic volumes (quermassintegrals) of Euclidean space can be restricted (as functionals on subsets) to arbitrary submanifolds, yielding intrinsically defined valuations known as the Lipschitz-Killing curvatures. We will explore the extendability of the Weyl principle to Finsler submanifolds of normed spaces, with the Holmes-Thompson intrinsic volumes replacing the Euclidean ones. Based on a joint work in progress with T. Wannerer.

Bounds for the volume ratio of convex bodies

Daniel Galicer
Universidad de Buenos Aires, Argentina

Given two convex bodies $K, L \subset \mathbb{R}^n$, the *volume ratio* between K and L is defined as

$$\mathrm{vr}(K, L) := \inf \left(\frac{|K|}{|T(L)|} \right)^{\frac{1}{n}},$$

where the infimum is taken over all affine transformations T such that $TL \subset K$ (here $|\cdot|$ stands for the n-dimensional Lebesgue measeure). Roughly speaking, this quantifies a measure relationship between the affine classes of the bodies involved (subject to the inclusion condition).

A natural question in convex geometry is the following: *Given a convex body K, how large is $vr(K, L)$ for arbitrary convex bodies $L \subset \mathbb{R}^n$?* Thus, it is interesting to study the *largest volume ratio* of K, defined as

$$\mathrm{lvr}(K) := \sup_{L \subset \mathbb{R}^n} \mathrm{vr}(K, L).$$

In this talk I will discuss the following sharp asymptotic general lower bound

$$\sqrt{n} \ll \mathrm{lvr}(K),$$

and overview the main ingredients of the proof (which is based on the probabilistic method). For many interesting classes we will see that the largest volume ratio behaves as the square root of the dimension of the ambient space.

Joint work with Mariano Merzbacher and Damián Pinasco.

Zhang's inequality for log-concave functions

Bernardo Gonzaléz Merino
Universad de Sevilla, Spain

Petty projection inequality (1971) states that among all the n-dimensional convex bodies the affine invariant quantity $|K|^{n-1}|\Pi^*K|$, where Π^*K denotes the polar projection body of K, is maximized by the Euclidean ball. Years later, Zhang (1991) showed that this affine invariant quantity is minimized only by simplices.

Petty projection inequality was successfully extended by Zhang (1999) to the space of Sobolev functions. In this talk we will prove a functional version of Zhang's inequality in the context of log-concave functions. Moreover, our inequality holds with equality if and only the function is of the form $ce^{-\|x\|_\Delta}$, for some simplex Δ containing the origin and some $c > 0$.

Randomized Urysohn–type inequalities

Thomas Hack
TU Wien, Austria
(joint work with P. Pivovarov)

As a natural analog of Urysohn's inequality in Euclidean space, Gao, Hug, and Schneider showed in 2003 that in spherical or hyperbolic space, the total measure of totally geodesic hypersurfaces that meet a given convex body K is minimized when K is a geodesic ball. We present a random extension of this result by taking K to be the convex hull of finitely many points drawn according to a probability distribution and by showing that the minimum is attained at uniform distributions on geodesic balls. As a corollary we obtain a randomized Blaschke–Santaló inequality on the sphere.

Intrinsic metric in spaces of compact subsets with the Hausdorff metric

Irmina Herburt
Politechnika Warszawska, Poland

If a metric space (X, ρ) can be endowed with an intrinsic metric ρ^* (the instrinsic distance of two points is defined as the infimum of the lengths of arcs joining these points), then the Hausdorff metric ρ_H in a space $\mathcal{C}(X)$ of compact subsets of X induces an intrinsic metric $(\rho_H)^*$, and the equality

$$(\rho_H)^* = (\rho^\star)_H$$

is satisfied. This implies that an isometry between spaces X_1 and X_2 with intrinsic metrics induces an isometry between $\mathcal{C}(X_1)$ and $\mathcal{C}(X_2)$ with intrinsic metrices.

The total curvature and Betti numbers of complex projective manifolds

Joseph Hoisington
University of Georgia, USA

We will prove an inequality between the total curvature of a complex projective manifold and its Betti numbers, and we will characterize the complex projective manifolds whose total curvature is minimal. These results extend the classical theorems of Chern and Lashof to complex projective space. We will also highlight some results about total curvature in compex projective space which have no analogue in the classical theory of submanifolds of Euclidean space.

A reverse Minkowski-type inequality

Daniel Hug
Karlsruher Institut für Technologie, Germany
(joint work with Károly Böröczky)

The famous Minkowski inequality provides a sharp lower bound for the mixed volume $V(K, M[n-1])$ of two convex bodies $K, M \subset \mathbb{R}^n$ in terms of powers of the volumes of the individual bodies K and M. The special case where K is the unit ball yields the isoperimetric inequality. In the plane, Betke and Weil (1991) found a sharp upper bound for the mixed area of K and M in terms of the perimeters of K and M. We extend this result to general dimensions by proving a sharp upper bound for the mixed volume $V(K, M[n-1])$ in terms of the mean width of K and the surface area of M. The equality case is completely characterized. In addition, we establish a stability improvement of this and related geometric inequalities of isoperimetric type.

Approximate Carathéodory's theorem in the PSD cone

Grigory Ivanov
Université de Fribourg, Switzerland

The distance between a point of the convex hull of a bounded set S and the set of all convex combinations of at most k points of S is a subject of approximate versions of Carathéodory's theorem. A famous problem of this type is an approximation of a John decomposition of the identity, which is a decomposition of the identity in $\mathbb{R}^n$ into a positive linear combination of rank one orthogonal projections. Recently, a major breakthrough was made by Batson, Spielman and Srivastava. They managed to show that it suffices to choose cn operators of a John decomposition to approximate the identity well enough. They improved Rudelson's theorem, in which the bound $cn \log n$ were proven. We discuss how to generalize Rudelson's theorem to the whole cone of positive semidefinite operators and show that the $\log n$ term cannot be omitted in a general case. This is joint work with Alexandr Polyanskii and Marton Naszodi.

On the maximal perimeter of sections of the cube

Hermann König
Universität Kiel, Germany

Keith Ball proved that central hyperplane sections of the n-dimensional cube perpendicular to $(1, 1, 0, \ldots, 0)$ have maximal $(n-1)$-dimensional measure. We show a similar result for the boundary of these sections, namely that the $(n-2)$-dimensional surface area (perimeter) of the n-dimensional cube is maximal for the hyperplane perpendicular to the vector $(1, 1, 0, \ldots, 0)$, answering a question of Pełczyński who had solved the three dimensional case. We study both the real and the complex versions of this problem. The result implies that the answer to an analogue of the Busemann-Petty problem for the surface area is negative in dimensions 14 and higher. This is joint work with A. Koldobsky.

Slicing and distance inequalities

Alexander Koldobsky
University of Missouri, USA

Slicing inequalities provide estimates for the volume of a body in terms of areas of its plane sections. We show new estimates of this kind depending on the (outer volume ratio) distance from the body to the class of intersection bodies or to the class of unit balls of subspaces of L_p. Many of the results hold for arbitrary measures in place of volume.

Probabilistic enumerative geometry

Antonio Lerario
Scuola Internazionale Superiore di Studi Avanzati (Trieste), Italy

Enumerative geometry deals with the problem of counting ("enumerating") geometric objects satisfying some constraint on their arrangement. For example: "how many lines in three-space intersect at the same time four given lines?" The answer is two if we are allowed to look for complex lines, but it depends on the four given lines if we search for real lines. In the complex framework this question (and similar) can be answered using a beautiful, sophisticated technique called Schubert calculus: it is the study of the way cycles intersect in complex Grassmannians. Unfortunately, over the reals this technique loses its power: this is the old problem of finding real solutions to real equations, for which the number of complex solutions only gives upper bounds. In this talk I will present a probabilistic approach to this problem, trying to address questions like: "how many lines in three-space intersect four given random lines?"

The answer to this question comes through the study of integral geometry in real grassmannians and has surprising connections to convex geometry and representation theory...

The existence of the extremals for polar Santaló inequalities

Ben Li
Tel Aviv University, Israel

The polar transform for geometric convex functions was rediscovered and studied by Artstein-Avidan and Milman. It is considered as the natural extension of the notion of polarity from convex bodies. Later on Artstein-Avidan and Slomka proved a functional version of Santaló inequality and its reverse for even geometric log-concave functions which are called polar Santaló inequalities. In this talk, we will discuss firstly a topological structure on the set of convex function of interest and then we show that the extremal cases in these inequalities can be indeed achieved.

Phase transisition phenomena in integral geometry

Martin Lotz
Warwick University, UK

The intrinsic volumes, divided by their sum, can be interpreted as discrete probability distributions. We present new concentration of measure results and variance bounds for various normalisations of the intrinsic volumes in Euclidean space. As a consequence, we obtain new interpretations of various classical formulas such as the Crofton Formula, the Principal Kinematic Formula, and expressions for rotation means and projections. The concentration results are based on an interpretation of the cumulant generating function in terms of integrals of concave and log-concave densities, and on new variance bounds for such densities. The results generalize previous work on the concentration of spherical intrinsic volumes by the authors, which was inspired by problems convex optimization and compressed sensing. This is ongoing joint work with Joel Tropp (Caltech), based on preliminary work with Mike McCoy, Ivan Nourdin, Giovanni Peccati and Joel Tropp.

Valuations on convex functions

Monika Ludwig
TU Wien, Austria

A function Z defined on a space of real-valued functions $\mathcal{F}$ and taking values in an Abelian semigroup is called a *valuation* if

$$\mathrm{Z}(f \vee g) + \mathrm{Z}(f \wedge g) = \mathrm{Z}(f) + \mathrm{Z}(g)$$

for all $f, g \in \mathcal{F}$ such that $f, g, f \vee g, f \wedge g \in \mathcal{F}$. Here $f \vee g$ is the pointwise maximum of f and g, while $f \wedge g$ is their pointwise minimum.

We discuss results on valuations defined on various spaces of convex functions. Classification theorems for $\mathrm{SL}(n)$ invariant valuations and the existence of a homogeneous decomposition for epi-translation invariant valuations are presented.

(Based on joint work with Andrea Colesanti and Fabian Mussnig)

Binet-Legendre construction

VLADIMIR MATVEEV
Friedrich–Schiller–Universität Jena, Germany

I explain how a simple integral construction from convex geometry solved many named problems in Finsler geometry. In partiucular we answer a question of Matsumoto about local conformal mapping between two Berwaldian spaces and use it to investigation of essentially conformally Berwaldian manifolds. We describe all possible conformal self maps and all self similarities on a Finsler manifold, generasing the famous result of Obata to Finslerian manifolds. We also classify all compact conformally flat Finsler manifolds. We solve a conjecture of Deng and Hou on locally symmetric Finsler spaces. We prove smoothness of isometries of Holder-continuous Finsler metrics. We construct new "easy to calculate" conformal and metric invariants of finsler manifolds.

SL(n) invariant valuations on convex functions

FABIAN MUSSNIG
TU Wien, Austria

Valuations on convex bodies have been of interest ever since they appeared in Dehn's solution of Hilbert's Third Problem in 1901. Two of the most fundamental valuations are the Euler characteristic and the n-dimensional volume and the first characterization of these operators as continuous, SL(n) and translation invariant valuations was obtained by Blaschke in the 1930s. Since then, many generalizations and improvements of his result were found. More recently, valuations on function spaces have been studied. We will present SL(n) invariant valuations on convex functions and corresponding characterization results. In particular, we will highlight similarities and differences with the theory of valuations on convex bodies, e.g. necessary conditions for a McMullen decomposition. Some of the presented results were obtained in joint work with Andrea Colesanti and Monika Ludwig.

Optimal comparison of weak and strong moments of random vectors with applications to concentration of log-concave measures and the theory of p-summing operators

PIOTR NAYAR
Uniwersytet Warszawski, Poland

We prove optimal, up to a universal constant, comparison of weak and strong moments of random vectors in arbitrary n-dimensional normed spaces. Our proof uses only basic linear algebra tools. We also discuss two applications of our result. First application concerns Latała-Wojtaszczyk concentration type bounds for log-concave measures in $\mathbb{R}^n$. Second application provides an upper bound on the so-called p-summing constant of an arbitrary finite dimensional Banach space.

An optimal plank theorem

Oscar Ortega Moreno
Warwick University, UK

We give a new proof of Fejes Tóth's zone conjecture: for any sequence $v_1, v_2, ..., v_n$ of unit vectors in a real Hilbert space H, there exists a unit vector v in H such that

$$| < v_k, v > | \geq \sin(\pi/2n)$$

for all k. This can be seen as sharp version of the plank theorem for real Hilbert spaces.s

The method we use is entirely different to the one in Jiang and Polyanskii paper "Proof of László Fejes Tóth's zone conjecture".

Almost Euclidean sections of convex bodies

Grigoris Paouris
Texas A&M University, USA

I will discuss few recent refinements of the classical theory of Dvoretzky which states that every convex body has a section of dimension $c(\varepsilon) \log n$ that is $(1+\varepsilon)$-Euclidean. My talk will be based on joint works with P. Valettas and K. Tikhomirov.

On critical values of the distance from a subset in the plane

Jan Rataj
Univerzita Karlova, Czech Republic
(joint work with Luděk Zajíček)

It is known that the set of critical values within some compact subinterval of $(0, \infty)$ of the distance function from a compact subset of the plane has Minkowski dimension at most $\frac{1}{2}$ (Fu, 1986). We show a stronger result: given any compact set $A \subset (0, \infty)$, the following two statements are equivalent: (i) there exists a compact planar set F such that any point of A is critical w.r.t. the distance from F; and (ii) if (I_i) are the bounded components of A^C then $\sum_i \sqrt{|I_i|} < \infty$. A related result removing the compactness assumption on A, as well as a variant in two-dimensional Riemannian surfaces, are also considered.

Anti-blocking, unconditional and beyond

Raman Sanyal
Goethe–Universität Frankfurt, Germany

A polytope in the first orthant is 'anti-blocking' if it contains the orthogonal projections onto coordinate subspaces of any of its points. From a combinatorial (optimization) point of view, these polytopes capture the combinatorics of hereditary set systems. In convex geometry, these polytopes are sometimes called 'convex corners' and are in one-to-one correspondence to 1-unconditional polytopes. In this talk I want to survey some of the many lives of anti-blocking polytopes, highlight the strong ties between combinatorics and geometry, and present old and new results relating to (mixed)volumes and lattice points.

Minimal volume product of convex bodies with various symmetries

Masataka Shibata
Tokyo Institute of Technology, Japan

Mahler's conjecture is one of the classical open problems in the area of convex geometry. It states that for a centrally symmetric convex body K in the n-dimensional Euclidean space, the product of the volume of K and that of the polar K° is greater than or equal to $4^n/n!$. This conjecture is still open for $n > 3$. It is natural to study the volume product of convex bodies with a suitable symmetry other than centrally symmetric one.

In this talk, we assume $n = 3$ and consider the volume product functional on the set of G-invariant three dimensional convex bodies, where G is a discrete subgroup of the orthogonal group $O(3)$. We will give sharp lower bounds of the functional for many cases. In particular, in the case where G is the alternating group A_4, the sharp lower bound gives a new partial result of non-symmetric Mahler conjecture for $n = 3$. We also discuss the high-dimensional case $n \geq 4$.

This talk is based on a joint work with Hiroshi Iriyeh.

Bounds for expectations of k-maxima of log-concave vectors

Marta Strzelecka
Uniwersytet Warszawski, Poland

We will present two-sided bounds for expectations of order statistics (k-th maxima) of moduli of coordinates of centred log-concave random vectors with uncorrelated coordinates. The bounds are exact up to multiplicative universal constants in the unconditional case for all k and in the isotropic case for $k \leq n - cn5/6$. We will also derive two-sided exact estimates of expectations of sums of k largest moduli of coordinates for some classes of random vectors. This generalize previously known results due to Gordon, Litvak, Schütt and Werner for vectors with weighted i.i.d. coordinates. Based on the joint work with Rafał Latała.

Refined concentration of measure derived from convex infimum convolution inequalities

Michał Strzelecki
Uniwersytet Warszawski, Poland

It is known since the seminal work of Talagrand in the '90s, that concentration properties of a probability measure on $\mathbb{R}^n$ may be considerably strengthened if we restrict the attention to convex Lipschitz functions. I shall discuss concentration of measure inequalities which can be derived from convex infimum convolution inequalities and present applications both to the Lipschitz and non-Lipschitz setting. Based on joint work with Radosław Adamczak.

On discrete Borell-Brascamp-Lieb type inequalities

Jesús Yepes Nicolás
Universidad de Murcia, Spain

If $f, g, h : \mathbb{R}^n \longrightarrow \mathbb{R}_{\geq 0}$ are non-negative measurable functions such that $h(x+y)$ is greater than or equal to the p-sum of $f(x)$ and $g(y)$, where $-1/n \leq p \leq \infty$, $p \neq 0$, then the classical Borell-Brascamp-Lieb inequality asserts that the integral of h is not smaller than the q-sum of the integrals of f and g, for $q = p/(np+1)$.

In this talk we will show a discrete analog of the above-mentioned result for the sum over finite subsets of the integer lattice $\mathbb{Z}^n$: under the same assumption as before, for $A, B \subset \mathbb{Z}^n$, we get

$$\sum_{x \in A+B} h(x) \geq \Big[\Big(\sum_{x \in \mathrm{r}_f(A)} f(x)\Big)^q + \Big(\sum_{x \in B} g(x)\Big)^q\Big]^{1/q},$$

where $\mathrm{r}_f(A)$ is obtained by removing points from A in a precise way, and depending on f. In particular, when considering certain discrete measures on $\mathbb{R}^n$, different Brunn-Minkowski type inequalities are derived. Among others, for the cardinality, we have that $|A+B|^{1/n} \geq |\mathrm{r}(A)|^{1/n} + |B|^{1/n}$.

When dealing with convex combinations one should however consider a different way of measuring (instead of using the cardinality). In this sense, we derive the following Brunn-Minkowski type inequality for the lattice point enumerator G_n, defined by $\mathrm{G}_n(M) = |M \cap \mathbb{Z}^n|$: given $\lambda \in (0,1)$ and non-empty bounded sets $K, L \subset \mathbb{R}^n$ such that $\mathrm{G}_n(K)\mathrm{G}_n(L) > 0$, we get

$$\mathrm{G}_n\big((1-\lambda)K + \lambda L + (-1,1)^n\big)^{1/n} \geq (1-\lambda)\mathrm{G}_n(K)^{1/n} + \lambda \mathrm{G}_n(L)^{1/n}.$$

We will show the necessity of adding the set $(-1,1)^n$ in the left-hand side of the above inequality as well as its functional counterpart. Moreover, we will show that both versions of the classical Borell-Brascamp-Lieb inequality (namely, for sums and convex combinations) for Riemann integrable functions can be derived as a consequence of these discrete versions.

This is about joint works with: David Iglesias López (University of Murcia) and Artem Zvavitch (Kent State University).